Assignment 1

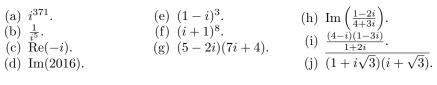
This homework is due Friday Jan 29.

There are total 40 points in this assignment. 35 points is considered 100%. If you go over 35 points, you will get over 100% for this homework (but not over 115%) and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper and give credit to your collaborators in your pledge. Your solutions should exhibit your work and contain full proofs. Bare answers will not earn you much.

This assignment covers Section 1.2 in Textbook.

(1) [10pt] Perform the required calculations and express your answer as a + bi (or just as a if the answer is a real number).



[*Hint* for item 1f: $A^8 = ((A^2)^2)^2$.]

(2) [10pt] Evaluate the following:

(a)
$$\overline{(1+i)(2+i)}(3+i)$$
.
(b) $(3+i)/(\overline{2+i})$.
(c) $\operatorname{Im}((1-i)^2)$.
(d) $\frac{1+2i}{3+4i} - \frac{4-3i}{2-i}$.
(e) $\operatorname{Re}((x-yi)^2)$.
(f) $\operatorname{Im}\left(\frac{1}{x-iy}\right)$.
(g) $\operatorname{Im}((x+iy)(-x+iy)$.
(h) $\operatorname{Im}((x+iy)^3)$.

- (3) [5pt]
 - (a) Show that $z\bar{z}$ is always a real number.
 - (b) Verify that $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$.
- (4) [10pt] Let $P(z) = a_n z^n + a_{n-1} z^{n-1} + \ldots + a_1 z + a_0$ be a polynomial of degree n.
 - (a) Suppose that a_n, \ldots, a_1, a_0 are all real. Show that if z_1 is a complex root of P then $\overline{z_1}$ is also a root. (In other words, non-real complex roots split into pairs of conjugates.) [*Hint:* Look at $\overline{P(z_1)}$.]
 - (b) Suppose polynomial $P = z^n + a_{n-1}z^{n-1} + \ldots + a_1z + a_0$ with complex coefficients has the above property, i.e. its non-real complex roots split into pairs of conjugates. Prove that all coefficients of P are, in fact, real. You can take for granted that *n*th degree polynomial has n complex roots¹ [*Hint:* If z_1, \ldots, z_n are roots of P, then P can be expressed as $P(z) = (z z_1)(z z_2) \cdots (z z_n)$.]
 - (c) (*Optional*, +3pt) Prove that every polynomial with real coefficients can be factored into a product of linear and quadratic polynomials with real coefficients. (As above, you can take for granted that *n*th degree polynomial has *n* complex roots.)

(5) [5pt] Consider matrices of the form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$ with a, b real. For $Z_1 = \begin{pmatrix} a_1 & b_1 \\ -b_1 & a_1 \end{pmatrix}$ and $Z_2 = \begin{pmatrix} a_2 & b_2 \\ -b_2 & a_2 \end{pmatrix}$ find Z_1Z_2 . Compare to multiplication of complex numbers $(a_1 + b_1i)(a_2 + b_2i)$. Make conclusions (I am not asking for an essay, one sentence is enough).

¹We will prove that later in the course.